Study of effect of Quantization on the signals and systems.

Objective

- Effect of quantization of signals
- Effect of quantization system
- Study of μ-law and A-law.

Introduction

In digital signal processing, **quantization** is the process of approximating a continuous range of values (or a very large set of possible discrete values) by a relatively-small set of discrete symbols or integer values.

A common use of quantization is in the conversion of a discrete signal (a sampled continuous signal) into a digital signal by quantizing. Both of these steps (sampling and quantizing) are performed in analog-to-digital converters with the quantization level specified in bits. A specific example would be compact disc (CD) audio which is sampled at 44,100 Hz and quantized with 16 bits (2 bytes) which can be one of 65,536 (i.e. 2^{16}) possible values per sample.

Mathematical description

The simplest and best-known form of quantization is referred to as scalar quantization, since it operates on scalar (as opposed to multi-dimensional vector) input data. In general, a scalar quantization operator can be represented as

$$Q(x) = g(\lfloor f(x) \rfloor)$$

where *x* is a real number to be quantized, $\lfloor \cdot \rfloor$ is the floor function, yielding an integer result $i = \lfloor f(x) \rfloor$ that is sometimes referred to as the *quantization index*, *f*(*x*) and *g*(*i*) are arbitrary real-valued functions.

The integer-valued quantization index *i* is the representation that is typically stored or transmitted, and then the final interpretation is constructed using *g*(*i*) when the data is later interpreted.

In computer audio and most other applications, a method known as *uniform quantization* is the most common. If *x* is a real-valued number between -1 and 1, a mid-rise uniform quantization operator that uses *M* bits of precision to represent each quantization index can be expressed as

$$Q(x) = \frac{\left\lfloor 2^{M-1}x \right\rfloor + 0.5}{2^{M-1}}$$

The value $2^{-(M-1)}$ is often referred to as the *quantization step size*. Using this quantization law and assuming that quantization noise is approximately uniformly distributed over the quantization step size (an assumption typically accurate for rapidly varying *x* or high *M*) and further assuming that the input signal *x* to be quantized is approximately uniformly distributed over the entire interval from -1 to 1, the signal to noise ratio (SNR) of the quantization can be computed as

$$\frac{S}{N_q} \approx 20\log_{10}(2^M) = 6.0206M \text{ dB}$$
.

From this equation, it is often said that the SNR is approximately 6 dB per bit.

In digital telephony, two popular quantization schemes are the 'A-law' (dominant in Europe) and 'μ-law' (dominant in North America and Japan). These schemes map discrete analog values to an 8-bit scale that is nearly linear for small values and then increases logarithmically as amplitude grows. Because the human ear's perception of loudness is roughly logarithmic, this provides a higher signal to noise ratio over the range of audible sound intensities for a given number of bits. u-LAW is given as,

$$F(x) = \operatorname{sgn}(x)\frac{\ln(1+\mu|x|)}{\ln(1+\mu)} \qquad -1 \leq x \leq 1$$

where μ = 255 (8 bits) in the North American and Japanese standards and A-LAW is

$$F(x) = \operatorname{sgn}(x)\begin{cases} \frac{A|x|}{1+\ln(A)}, & |x| < \frac{1}{A} \\ \frac{1+\ln(A|x|)}{1+\ln(A)}, & \frac{1}{A} \leq |x| \leq 1 \end{cases},$$

where A=87.6.

Experiment 1 - Quantization of Sinusoid Signals

- Generate a sine wave of frequency of 5Hz, sampled at 1000Hz. And plot it.
- Use 10,8,6,4,3 bits to represent the values of the sine wave (uniform quantizer). And plot it
- Calculate the error obtained by using different bits.
- Repeat the experiment by sampling the signal at 10MHz.
- Repeat the experiment by adding gaussian noise to the sine wave and plot the relation between quantization error as a function of SNR.
- Comment on the results.

Experiment 2- Coefficient Quantization

1. Consider the filter

$$H(z) = \frac{1 - 2.1872z^{-1} + 3.0055z^{-2} - 2.1872z^{-3} + z^{-4}}{1 - 3.1912z^{-1} + 4.1697z^{-2} - 2.5854z^{-3} + 0.6443z^{-4}}$$

- Plot the magnitude frequency response in dB, and pole zero plot of the filter.
- The transfer function can be factored as follows, where the poles nearest the unit circle and the zeros close to those poles appear in the second term:

$$H(z) = \left(\frac{1 - 0.6511z^{-1} + z^{-2}}{1 - 1.5684z^{-1} + 0.6879z^{-2}}\right)\left(\frac{1 - 1.5321z^{-1} + z^{-2}}{1 - 1.6233z^{-1} + 0.9366z^{-2}}\right)$$

- Plot the magnitude frequency response in dB, and pole zero plot of each part of the filter and then the complete response.
- Represent the coefficients using 4 bits. Plot the magnitude response in dB of both representations. Also plot pole-zero.
- Repeat the experiment with 2 and 4 bits and comment on the result.

Experiment 3.

- Generate a sine wave of frequency of 5Hz, sampled at 1000Hz.
- Assume μ = 255, plot $F(x)$ as function of x.
- Repeat the experiment for μ = 7, 15, 31, 127.
- Assume A=87.6 plot $F(x)$ as function of x.
- Repeat the experiment for A= 1, 2, 5, 10, 50.
- Give the binary represent of the signal in each case with 8 of bits. With μ = 255 and A=87.6 .
- Compute the error in each case and compare the results with uniform quantization.
- Can we retrieve the signal back? Try that!!
- Comment on the results.

Experiment 4.

- Use the music.wav file to study the effect of quantization.
- Read the file in matlab. (use wavread)
- Study effect of uniform qunatization on this piece of music by using 3, 4 … 10 bits. (use wavwrite). Observe by listening the music.
- Observe and comment on the results after companding.