Study of effect of Quantization on the signals and systems.

Objective
• Effect of quantization of signals
• Effect of quantization system
• Study of μ-law and A-law.

Introduction

In digital signal processing, quantization is the process of approximating a continuous range of values (or a very large set of possible discrete values) by a relatively-small set of discrete symbols or integer values.

A common use of quantization is in the conversion of a discrete signal (a sampled continuous signal) into a digital signal by quantizing. Both of these steps (sampling and quantizing) are performed in analog-to-digital converters with the quantization level specified in bits. A specific example would be compact disc (CD) audio which is sampled at 44,100 Hz and quantized with 16 bits (2 bytes) which can be one of 65,536 (i.e. $2^{16}$) possible values per sample.

Mathematical description

The simplest and best-known form of quantization is referred to as scalar quantization, since it operates on scalar (as opposed to multi-dimensional vector) input data. In general, a scalar quantization operator can be represented as

$$Q(x) = g\left(\lfloor f(x) \rfloor\right)$$

where $x$ is a real number to be quantized, $\lfloor \cdot \rfloor$ is the floor function, yielding an integer result $i = \lfloor f(x) \rfloor$ that is sometimes referred to as the quantization index, $f(x)$ and $g(i)$ are arbitrary real-valued functions.

The integer-valued quantization index $i$ is the representation that is typically stored or transmitted, and then the final interpretation is constructed using $g(i)$ when the data is later interpreted.

In computer audio and most other applications, a method known as uniform quantization is the most common. If $x$ is a real-valued number between -1 and 1, a mid-rise uniform quantization operator that uses $M$ bits of precision to represent each quantization index can be expressed as

$$Q(x) = \frac{\lfloor 2^{M-1}x \rfloor + 0.5}{2^{M-1}}$$

The value $2^{-(M-1)}$ is often referred to as the quantization step size. Using this quantization law and assuming that quantization noise is approximately uniformly distributed over the quantization step size (an assumption typically accurate for rapidly varying $x$ or high $M$) and further assuming that the input signal $x$ to be quantized is
approximately uniformly distributed over the entire interval from -1 to 1, the signal to noise ratio (SNR) of the quantization can be computed as

$$\frac{S}{N_q} \approx 20 \log_{10}(2^M) = 6.0206M \text{ dB}$$

From this equation, it is often said that the SNR is approximately 6 dB per bit.

In digital telephony, two popular quantization schemes are the 'A-law' (dominant in Europe) and 'μ-law' (dominant in North America and Japan). These schemes map discrete analog values to an 8-bit scale that is nearly linear for small values and then increases logarithmically as amplitude grows. Because the human ear's perception of loudness is roughly logarithmic, this provides a higher signal to noise ratio over the range of audible sound intensities for a given number of bits. μ-LAW is given as:

$$F(x) = \text{sgn}(x) \frac{\ln(1 + \mu|x|)}{\ln(1 + \mu)} - 1 \leq x \leq 1$$

where $\mu = 255$ (8 bits) in the North American and Japanese standards and A-LAW is

$$F(x) = \text{sgn}(x) \begin{cases} \left( \frac{A|x|}{1 + \ln(A)} \right), & |x| < \frac{1}{A} \\ \left( \frac{1 + \ln(|x|)}{1 + \ln(A)} \right), & \frac{1}{A} \leq |x| \leq 1, \end{cases}$$

where $A=87.6$.

Experiment 1 - Quantization of Sinusoid Signals

- Generate a sine wave of frequency of 5Hz, sampled at 1000Hz. And plot it.
- Use 10,8,6,4,3 bits to represent the values of the sine wave (uniform quantizer). And plot it.
- Calculate the error obtained by using different bits.
- Repeat the experiment by sampling the signal at 10MHz.
- Repeat the experiment by adding gaussian noise to the sine wave and plot the relation between quantization error as a function of SNR.
- Comment on the results.

Experiment 2- Coefficient Quantization

1. Consider the filter

$$H(z) = \frac{1 - 2.1872z^{-1} + 3.0055z^{-2} - 2.1872z^{-3} + z^{-4}}{1 - 3.1912z^{-1} + 4.1697z^{-2} - 2.5854z^{-3} + 0.6443z^{-4}}$$
• Plot the magnitude frequency response in dB, and pole zero plot of the filter.
• The transfer function can be factored as follows, where the poles nearest the unit circle and the zeros close to those poles appear in the second term:

$$H(z) = \left(\frac{1 - 0.6511z^{-1} + z^{-2}}{1 - 1.5684z^{-1} + 0.6879z^{-2}}\right)\left(\frac{1 - 1.5321z^{-1} + z^{-2}}{1 - 1.6233z^{-1} + 0.9366z^{-2}}\right)$$

• Plot the magnitude frequency response in dB, and pole zero plot of each part of the filter and then the complete response.
• Represent the coefficients using 4 bits. Plot the magnitude response in dB of both representations. Also plot pole-zero.
• Repeat the experiment with 2 and 4 bits and comment on the result.

Experiment 3.
• Generate a sine wave of frequency of 5Hz, sampled at 1000Hz.
• Assume $\mu = 255$, plot $F(x)$ as function of $x$.
• Repeat the experiment for $\mu = 7, 15, 31, 127$.
• Assume $A=87.6$ plot $F(x)$ as function of $x$.
• Repeat the experiment for $A= 1, 2, 5, 10, 50$.
• Give the binary represent of the signal in each case with 8 of bits. With $\mu = 255$ and $A=87.6$.
• Compute the error in each case and compare the results with uniform quantization.
• Can we retrieve the signal back? Try that!!
• Comment on the results.

Experiment 4.
• Use the music.wav file to study the effect of quantization.
• Read the file in matlab. (use wavread)
• Study effect of uniform quantization on this piece of music by using 3, 4 … 10 bits. (use wavwrite). Observe by listening the music.
• Observe and comment on the results after companding.